

TRIGONOMETRIC INTEGRALS. PART VII

Integrals of the form:

$$\int \sin(mx) \cos(nx) dx \quad \int \sin(mx) \sin(nx) dx$$

$$\int \cos(mx) \cos(nx) dx$$

Ex: $\int \sin(5x) \cos(3x) dx$

Ex: $\int \sin(3x) \sin(4x) dx$

Ex: $\int \cos(2x) \cos(5x) dx$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

Ex): $\int \sin(5x) \cos(3x) dx = \frac{1}{2} \int [\sin(2x) + \sin(8x)] dx$

$$= \frac{1}{2} \int \sin(2x) dx + \frac{1}{2} \int \sin(8x) dx$$

$$u = 2x \\ du = 2 dx$$

$$w = 8x \\ dw = 8 dx$$

$$= \frac{1}{2} \cdot \frac{1}{2} \int \sin(2x) 2 dx + \frac{1}{2} \cdot \frac{1}{8} \int \sin(8x) 8 dx$$

$$= \frac{1}{4} \int \sin u du + \frac{1}{16} \int \sin w dw$$

$$= -\frac{1}{4} \cos u - \frac{1}{16} \cos w + C = -\frac{1}{4} \cos(2x) - \frac{1}{16} \cos(8x) + C$$