

TWO-PROPORTION Z-TEST

One proportion

$$H_0: p \leq p_0 \quad H_0: p \geq p_0 \quad H_0: p = p_0$$

$$H_a: p > p_0 \quad H_a: p < p_0 \quad H_a: p \neq p_0$$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$

two proportions

$$p_1 > p_2$$

$$p_1 < p_2$$

$$p_1 \neq p_2$$

$$H_0: p_1 - p_2 \leq 0$$

$$H_0: p_1 - p_2 \geq 0$$

$$H_0: p_1 - p_2 = 0$$

$$H_a: p_1 - p_2 > 0$$

$$H_a: p_1 - p_2 < 0$$

$$H_a: p_1 - p_2 \neq 0$$

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}} = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p} \hat{q} \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$\hat{p} = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2}$$

$$\hat{q} = 1 - \hat{p}$$

EX: Given the following data, do a hypothesis test to determine if $P_2 > P_1$

$$n_1 = 300$$

$$x_1 = 200$$

$$n_2 = 400$$

$$x_2 = 290$$

$$P_2 > P_1 \Rightarrow P_2 - P_1 > P_1 - P_2 \Rightarrow 0 > P_1 - P_2$$

$$H_0: P_1 - P_2 \geq 0$$

$$H_a: P_1 - P_2 < 0$$

$$\hat{P}_1 = \frac{x_1}{n_1} = \frac{200}{300} \approx .67$$

$$\hat{P}_2 = \frac{x_2}{n_2} = \frac{290}{400} = .725$$

$$\hat{P} = \frac{200 + 290}{300 + 400} = \frac{490}{700} = .7$$

$$Z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P} \hat{Q} \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} = \frac{(.67 - .725)}{\sqrt{.7 \times .3 \left(\frac{1}{300} + \frac{1}{400} \right)}}$$

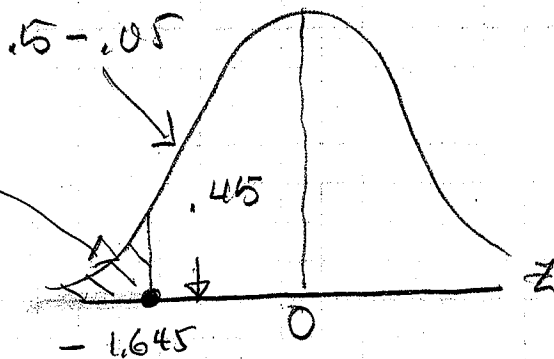
$$Z = -1.57$$

$$\alpha = .05$$

$$RR: Z < -1.645$$

Decision:

Fail to reject H_0



$$\hat{p}_1 = \frac{200}{300} = \frac{2}{3}$$

$$\bar{z} = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{\frac{2}{3} - .725}{\sqrt{.7 \times .3 \left(\frac{1}{300} + \frac{1}{400}\right)}}$$

$$\approx -1.67$$