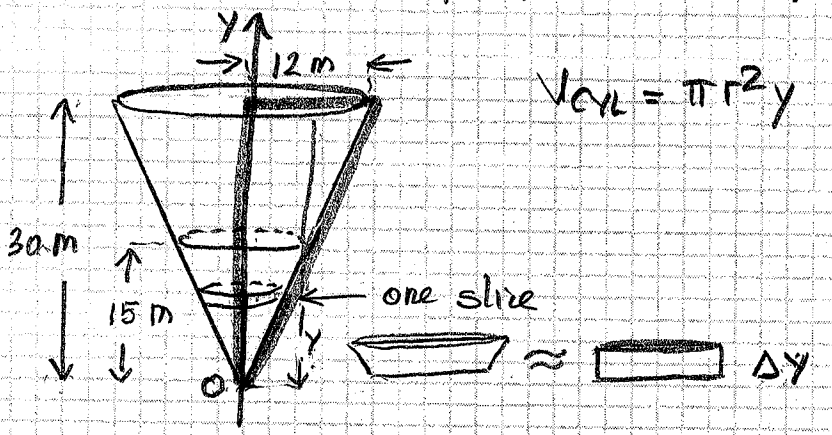
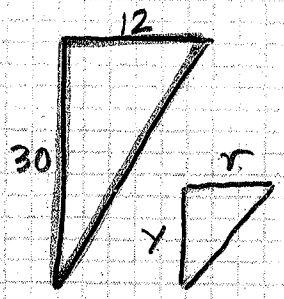
WORK: PUMPING WATER OUT OF A CONICAL TANK

An inverted conical tank of height 30 m and radius 12 m is filled with water to a depth of 15 m. How much work is needed to pump all the water out the top of the tank?



$V_{CYL} = \pi r^2 y$



$\frac{r}{y} = \frac{12}{30} \Rightarrow$

$\Rightarrow r = \frac{2}{5} y$

$V_{\text{1 slice}} = \pi \left(\frac{2}{5} y\right)^2 \Delta y \quad m^3$

$\rho_{WATER} \approx 10^3 \ kg/m^3 \qquad \rho = \frac{m}{V} \Rightarrow m = \rho \cdot V$

$m = 10^3 \frac{kg}{m^3} * \pi \frac{4}{25} y^2 \Delta y \ m^3 = 160 \pi y^2 \Delta y \ kg$

$F = m \cdot a \qquad F_g = 160 \pi y^2 \Delta y \ kg * 9.8 \frac{m}{s^2} =$

$F_g = 1568 \pi y^2 \Delta y \ N \qquad W = F \cdot d$

$W_{\text{1 slice}} = 1568 \pi y^2 \Delta y \ N * (30 - y) \ m \qquad N \times m = \text{Joule}$

$W_{total} = \int_0^{15} 1568 \pi y^2 (30 - y) \, dy \quad \text{Joules}$

$= 1568 \pi \int_0^{15} (30y^2 - y^3) \, dy = 1568 \pi \left(30 \frac{y^3}{3} - \frac{y^4}{4}\right) \Big|_0^{15}$